*Note: accuracy of these solutions cannot be guaranteed – feel free to comment or fix any errors you may see. Also, it is recommended to use the desktop version of Word due to the equations (from the web version, File -> Info).*

# Answer 1

## Part a

. After all, the variables are iid, which means that the variance of the estimator is . The result follows immediately. The proof for the variance is also in the notes.

## Part b

The variance is . This is clearly lower than the expected return , and hence the answer is yes.

## Part c

Not quite. The main issue with this approach is that the correlation between any two random variables is no longer 0! Also see <https://edstem.org/us/courses/14707/discussion/949082>.

Note: there is a fault in the screenshot. It should be , not .

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## Part d

This is also in the notes. The below screenshot is cited from https://edstem.org/us/courses/14707/discussion/949082?answer=2155476

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# Answer 2

## Part a

*I suspect there may be multiple right answers for this one. The simplest answer would be simply a call option, but the issue is that it may not meet the “combination” requirement of the question (citing from* [*https://edstem.org/us/courses/14707/discussion/958369?answer=2174477*](https://edstem.org/us/courses/14707/discussion/958369?answer=2174477)*). Hence, suitable answers would be a bull spread (which while answers the question, limits the upper bound), or simply buying a call with price and selling a put with price , where . Notice that as the buyer of a put will prefer lower prices, in a bullish case the option will not be exercised and hence benefits (up to a bound) increasing prices, with the sum of the two permitting an unbounded upper limit. The payoff diagram can hence be drawn similarly.*

## Part b

Use put-call parity? Indeed.

Put-call parity states that given the call price *C*, put price *P* and the amount lent and stock price *S*, we have

Now, what do we have? We have *C*. We also know what the price of the dividend is – *D*. **NB**: I am not sure whether *D* refers to the present stream or is meant to be the sum of all dividend payments already (*think* it’s the former, but I’m not very confident)

Then, the value of the put option is simply

Where *r* is the discount factor. Again, I am **not** sure of the correctness of this one and it’s likely there are bugs in my reasoning.

## Part c

The first part of this question is bookwork, and the relevant part of the slides is reproduced:

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… which means that American call options reduce to a European option, hence making their option prices the same.

The second part of the answer is yes. The reason is that from the lectures, we know that it is optimal to exercise an American put early, because of time value. This implies that one has the added leverage of being *able* to exercise early if optimal, which is not possible for a European option. Hence that’s indeed more valuable (and costlier as a result).

# Answer 3

## Part a

Simply divide each of the five years in the 3% coupon bonds into five one-year zero-coupon bonds. Note however that the rates will change as technically we will need to discount for the year.

Alternate: simply divide into five zero-coupon bonds, with 1, 2, 3, 4 and 5 years respectively. In this case the discounting is “built-in”.

## Part b

Let the price of the bond be *P* and principal be *F*. Then from the formulas given in the lecture (this can also be derived intuitively given that this is a zero-coupon bond):

It’s assumed that “duration” refers to the Macaulay duration. Again using the formulas,

(indeed this is as obvious as it sounds because the zero coupon bond has an automatic duration at maturity)

For convexity, it is similar, We know that

Take the second derivative:

Substitute:

## Part c

Todo

Yield of 1 year zero coupon bond: 7.5%

Yield of 2 year zero coupon bond: 10.43%

Yield of 3 year zero coupon bond: 10.56%

Forward rate 1-2 : 13.4%

Forward rate 2-3: 10.8%

N.B: one can show an arbitrage argument by showing that the forward rate *f* cannot be anything other than . For instance, if *f* is too low, one could buy a 2-year bond and sell a one-year bond with rate and one with rate *f* after a year – after two years you’ll be left with free money = Type B arbitrage!

## Part d

Help pls. +1 brilliant thanks mate

Working on it (update: done)

Thanks man

Genius

The first part of this question is interesting. One option is to use Lagrangians – the solution of this EdSTEM post (https://edstem.org/us/courses/14707/discussion/941073?answer=2153036) is referenced below.

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Chart, scatter chart

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Diagram

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I had also considered using matrix calculus; unfortunately, that does not seem to work as elegantly as in the first coursework as only one of the constraints can be combined (while we have two).

*Part 2 <3*

Waltz through the variance given by taking the derivative:

and set them to 0. This gets us

(Important – they may look like, but aren’t matrices, so an answer of is *not* valid and will send you through an ugly rabbit hole!)

Then, fit it back in the equation:

We’re not done yet! To complete the very last part, let’s take a weight *w* in the optimal portfolio T and in an arbitrary portfolio *q*. Then, note that

We want to minimise this. Take the derivative with respect to the weights:

But then considering that *T* is the optimal portfolio, we must actually allocate all the weights to it (so *w* = 1) as we want the minimum variance. That is,

This completes the proof. Disclosure: page 15 of [th8is site](https://teach.business.uq.edu.au/courses/FINM6900/files/module-1/notes/CAPM.pdf) helped me to do it.